A multi-agent cognitive framework to model human decision making under bounded rationality

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Abstract

Classical Decision Theory has been widely used in multi-agent systems, but are not representative of decision-making in real conditions. So, we built an architecture based on several cognitive psychological theories (e.g. Simon's decision theory, Tversky's model of elimination by aspect, etc.) to take human bounded rationality into account. We adopt an intermediate-level of cognitive modelisation, situated between logical level and physical level: the cognition is viewed as modular. In the CODAGE model, the decision maker is modeled by a multi-agent system, where each agent represents a particular sub-process of the whole decision. This framework permits the implementation of cognitive heuristics leading to biaised decision. We illustrate this approach with a simulation of a small experimental financial market, for which our model was able to replicate some human decision behaviors.

1 Modelize realistic decision-making for simulation purpose

1.1 A model is created for a given purpose

In a broad range of domains, one tries to describe social systems [Axtell, 1996, Gilbert, 1994] as microeconomical systems, consumer populations or firms. The multi-agent paradigm [Ferber, 1999, Wooldridge, 2002] allows to create models of societies composed by autonomous entitites (the agents) interacting within a common environment. A model is created for a specific purpose : to simulate a real system (for a better understanding of this system or for use as decision-support) or to study a purely theoretical one. These approaches are very different and have to be well distinguished. They don't need the same methodology and can't be used to infer the same conclusions. For the second purpose, when studying an ideal economic system, the decision-making system of the agents can be based on a normative model. But, for modelizing an actual society, we have to find a descriptive model for the agents. To give this model explanatory capabilities, we need to represent the process which are really used by humans [Edmonds and Moss, 2001].

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1.2 What is decision-making ?

One can define decision-making as *the process of selecting a course of action from among multiple alternatives*. According to classical decision-making, the decision-maker is said to evaluate these alternatives according to several criteria. Aggregative models decompose the decision-making process in three steps: (1) determine the utility of each alternative, (2) maybe include the uncertainty and probability informations, and (3) choose the best alternative on the basis of these utilities. However, evidence from a lot of experiments proove that the actual decisions of human are *biaised*. Due to cognitive limitations, human cannot represent and evaluate all the alternatives (Simon, [Simon, 1955]). The biais and heuristics research (Kahneman, Slovic and Tversky [Kahneman et al., 1982]) has listed some heuristics used during the perception, representation and selection processes. Elimination By Aspects (Tversky & Kahneman [Tversky, 1972]), the *satisfycing* solution paradigm (Simon [Simon, 1955]) or Probabilistic Mental Models ([Gigerenzer and Goldstein, 1996]) also explain how the alternatives can be compared with a computational model (and not an aggregative one).

All these descriptive models focus on the selection process between several alternatives. But they explain neither how these alternatives are build nor how the environment is perceived and represented. Several authors [Simon, 1955, Brunswik, 1952] underline the relation between this decision process and the environement itself. According to this point of view, the decision making process could be defined as *the cognitive process of reaching a decision*. The first step, *perception*, is selective and imperfect. It provides raw and complex informations that cannot be used in this form. The decision-maker has to *integrate* this data, i.e. to make it into the decision-maker frame of reference. Then the alternatives are built. If the decision-maker is not given any external choice, he has to provide himself this alternatives (case of the chess player) based on personal knowledge.

However, is it relevant to see decision-making as a unique kind of process ? Prior studies show that several strategies can be used: case-based reasoning [Klein, 1993], analytical analysis and so. Classical Decision-Making distinguish the expert decision-maker (opposed to the naive decision-maker), which has accumulated knowledge about his domain and uses adapted methods for solving problems. Decision-making is also influenced by investment in a task: if the decision-maker has to find a perfect decision (expert who has to justify his choice, for instance) he will invest a lot of cognitive ressources. But if the consequences of the choice aren't risky, as for a consumer buying fruits, he will adopt a rapid and cognitivly costless strategy. For explaining this strategy choice, few models have been proposed, as the Cognitive Continuum Theory [Engel et al., 1995] which provides a complexity scale for ranking the different strategies according to their complexity.

2 The CODAGE Architecture

CODAGE abords decision-making in its broader definition. So, we described the decision-making process with three main phases: perception, alternatives building and choice. These steps are concurrent : as soon as an alternative is built, it can be choosed as definitive decision. We also claim that some mechanisms are pervasive, as the anchoring process: in trading, round values (e.g. 50) are easier to memorize and might be favored as decision thresholds. This number anchoring effect will not only bias the perception process, but also the alternative choice. Given our concurrency mechanism, we adopt the "Minsky's Society of Mind" point of view [Minsky, 1986] and propose to model all these subprocess modules as autonomous agents in interaction. This agents working at cognitive level, the *micro-agents*, are specialized autonomous entities which interact at different phases of the decision process. They work on a shared *tree of alternatives* implemented as a blackboard system to facilitate information sharing, as depicted in Figure 2 below.

CODAGE implements the decision process in a dynamic and non-linear way. During the alternatives building, the alternatives are integrated and evaluated. So, despite the alternatives are not yet all build and are still partial, the evaluation of these alternatives can already eliminate or find a satisfycing alternative. This concords with observations that different presentations of the same alternatives conducts to different decisions (Tversky and Kahneman [Tversky and Kahneman, 1986]). The CODAGE architecture is composed by *micro-agents* working in a multi-threaded way [Kant and Thiriot, 2006]. A micro-agent represents a part of the decision process: for instance, there is a perception agent which performs the perception step, agents working on the intergration phasis and an agent which implements the choice between the alternatives. This modular representation permits a more intuitive understanding; but as any parallel system, it can also be implemented as a linear computational process that simulates this parallelism.

CODAGE is able to represent complex informations (typed values, symbols), to implement elementary reasoning process if necessary, to insert informations during conception and to implement a non-aggregative alternative comparison. CODAGE is neither a computational theory of mind nor a theory of cognition. It is a conceptual framework in which we can represent this given steps for modelization purpose. At this time, this model has been applied to model an experimental market and we are now adapting it for customer behaviour simulation.

2.1 Knowledge representation

What kind of knowledge representation is suitable for a decision-making model ? For judgment, Brunswick proposed to discretize the informations in elementary signals or cues [Hammond et al., 1975]. The set of cues is aggregated for calculating the final strength of the judgment. At a more cognitive level, Anderson proposed a cognitive architecture based on both declarative and procedural knowledge [Anderson, 1983]. But this architecture does not allow to easily include heuristics, such as anchoring[1], nor to implement case-based reasoning. For a social simulation purpose, this model is too constrained by the representation level. We choosed an intermediate modelisation level in which knowledge can be symbolic, procedural or numerical. For instance in a trading game, `$capital[capital_euros]=2501.2` means that the attribute "capital" has a value of 2501.2 and this numerical value is typed as "capital_euros"; `buy_proposition(alice,3,14.5)` encodes the fact that Alice proposed to buy 3 stocks at 14.5 euros each. We add two important mechanisms to encode the information processing prescribed by our cognitive model : salience and tree of alternatives.

[1] This as been done in [Petrov and Anderson, 2000], but it is not easy to implement this in interaction with other tasks.

2.1.1 Salience

The *salience*[2] of a fact represents its importance within the selective attention process. Each micro-agent ma of global agent pool $\mathcal{P}$ can vote to set the salience of a knowledge K within an alternative C (context, i.e. a possible state of the world). We denotes $v_{ma,K,C} \in [0,1]$ the resulting value of such a vote. If the value is strictly positive, K is added to C with the corresponding salience value $v_{ma,K,C}$ if K is new to C ; if K is already instantiated in C, then its value is simply updated in the equation 1 that gives the final value $S_{K,C}$ of the salience of a given knowledge K within the context of an alternative C as the mean of the micro-agents ' votes:

$$S_{K,C} = \frac{\sum_{ma \in \mathcal{P}} v_{ma,K,C}}{card(\mathcal{P})} \quad (1)$$

Neurobiology supposes that a salient fact is processed more quickly than an non-salient one [Berthoz, 2003]. In our model, knowledge-source agents will focus their attention on salient facts. This is implemented with two kinds of delays: an *event propagation delay* $d_{K,C}$, which causes agents to be warned later for non-salient facts, and a *reaction delay* $d_{R,C}$ for each rule R activable in a knowledge-source agent.

The propagation delay is 0 if the salience is 1, and rises to a maximum level (γ) if the salience is 0. We use the following function:

$$d_{K,C} = -\gamma . \left(\frac{S_{K,C} - 1}{S_{K,C} + 1} \right) \quad (2)$$

Figure 1 shows how this delay evolves in function with salience.

Let $A_{R,C}$ be the activation of a rule R in alternative context C. It equals the mean of premise 's saliences:

$$A_{R,C} = \frac{\sum_{\pi \in Premises(R)} s_{\pi,C}}{card(Premises(R))} \quad (3)$$

The agent reaction delay is inspired by the ACT-R theory [Anderson et al., 2004, p.1043]:

$$d_{R,C} = I + Fe^{-A_{R,C}} \quad (4)$$

where $I = 597ms$ and $F = 890ms$.

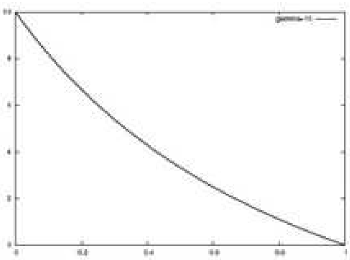

Figure 1: Propagation delay as a function of salience ($\gamma = 10$)

Finally, the total information propagation delay of knowledge K in context C is given by:

$$td_{K,C} = d_{K,C} + d_{R,C} = -\gamma . \left(\frac{S_{K,C} - 1}{S_{K,C} + 1} \right) + I + Fe^{-A_{R,C}} \quad (5)$$

Our mechanism of propagation delay has two major benefits:

[2] Several psychological studies support the concept of salience. Due to lack of space, we suggest this review of salience effects [Haynes and Kachelmeier, 1998]

- the system is more robust to information permutation: even if a low-salient information is added prior to a high-salient one, the latter will be considered first.
- it enables the partial exploration of the tree of alternatives, since the micro-agents will act based upon the most salient facts. Alternatives based on low-salient facts will not lead to further consequences exploration.

2.1.2 Tree of Alternatives (TA)

In CODAGE, the current state of decision (built alternatives, expected effects of the decision) is encoded into a decision tree[3], as the one depicted in Figure 3. Each node is an alternative that represents a possible state of the world (past, current or future). TA is a decision tree, as in decision theory, but it will be only partially built and explored to be consistent with bounded rationality. TA works at a symbolic level: each alternative represents an instantiation context in which each micro-agent may add a fact and/or an action into the tree: this is a way to share information between micro-agents. Each fact in the tree has a salience that measure its degree of importance.

Arcs between alternatives nodes represent *transitions* in time, that what produce the transition from one alternative (parent) to another one (child). We implemented two types of transitions that triggers the change to a new state of world:

- *action* transition: a possible action, performed by the macro-agent
- *fact* transition: the probability that some attribute will have a certain value (e.g. the final stock value will be 56.2 Euros at the closing of the market) or that an other agent perform some action (e.g. bob has sold 5 stocks to alice at 14.6 Euros)

TA could be viewed as a *blackboard* system. As one knows, the opportunistic control of knowledge sources (the micro-agents in our case), that is running the right agent on the right data at the right time, is a tricky issue in blackboards [Corkill, 2003]. In our model, there is no fixed agenda to select one agent at a time: each agent is autonomous, and is able to modify data on the decision tree whenever it needs to. From a computer implementation point of view, it is a full multi-threading process. To preserve data coherence and integrity inside the tree, we implemented a mechanism to *solve eventual contradictions*. Any agent can signal a contradiction inside a given context C. In this case, the blackboard removes the two contradictory facts from C, creates two children of C (C_1 and C_2) and instantiates the two incompatible facts in two separated contexts. This method preserves the existence of the two solutions while avoiding the contradictions.

2.2 Agents

Each agent encodes a subprocess of the decision system, like an heuristic, an inference mechanism, a knowledge database, perception, etc.

The *perception agent* (abbreviated as **PER** in the remaining part of this paper) imports informations from environment: e.g. buy and sell orders, accepted transactions and so. This knowledge is introduced at the root of the TA as symbols, predicates and variables. Initial

[3]We do not assume that a human decision-maker actually has such a decision tree inside his/her head. This is just a convenient modeling tool to tackle alternatives management in our model.

salience values are set, depending on decision maker's habits and experience (what he/she is used to consider as important information)[4].

The *egocentric agent* (**EGO**) helps the macro-agent to selectively enhance the salience on every facts and actions he/she is involved in (e.g. the orders he gave, the proposals he made).

The *world rules agent* (**WRU**) contains the knowledge about the world rules. It encodes the main rules and constraints within the environment like the possible actions (e.g. in our simulated game, a trader can emit buy or sell order, or cancel a previous order), the forbidden actions (e.g. to buy with a null capital), and some anticipated consequences of actions (e.g. if an order is accepted, capital and bids count are updated according to a particular formula).

The *expertise agent* (**EXP**) contains a set of domain-specific heuristics and strategies the decision maker may use to perform his/her actions. In our example of a trading game, these strategies will increase the salience of critical attributes like total capital, gain and loss. They will give the relevant hypothesis to explore, like buying or selling a share. They also value the different facts (e.g. in term of expected outcomes).

The *anchoring agent* (**ANC**) gives the set of anchoring values, that will be used as reference points. In a predicate where some attribute value is unknown, the anchoring agent enumerates all possible values, and will propose to anchor to an already perceived value or to a given reference-point value, e.g. a value linked to the personal situation of the decision maker, or a constant specific to the problem domain (a national interest rate for instance).

The *uncertainty agent* (**UNC**) encodes the uncertainty of informations in the TA. It (i) sets probability p_K for a fact K to occur, and (ii) sets the probability $Pr(C))$ of alternative context C to occur in the real world.

The *decision agent* (**DEC**) monitors the decision tree and implements the search for the satisfycing solution. When an alternative is added into the tree, it evaluates it. If this is a *immediately satisficing solution*, the tree building process is stopped, and the action that created this branch is selected. If the alternative is too low (the aggregated utility of this alternative is lower than an elimination threshold), it is ignored. In other cases, the alternative is considered to be studied later, and added to an internal list. When this list is full, the alternative having the highest aggregated utility is selected. We compute the utility of an alternative A as follows :

$$AU(A) = f(\sum_{C \in Child(A)} Pr(C).u(C)) \tag{6}$$

where $Child(A)$ is a the set of immediate children of A in the tree. $Pr(C)$ the probability given by UNC agent (see above) and f is an utility normalization function, a numerical function valued in $[0,1]$. For instance, we could adopt CARA (Constant Absolute Risk Aversion) function for risk-averse subjects $f(x) = -(1/\rho).e^{-\rho.x}$, where $\rho \in [0,1]$ is a risking factor.

The utility $u(C)$ of an alternative C is given using a classical multi-attribute utility model, where we use the salience to weight each fact :

$$u(C) = \sum_{K \in C} p_K.v(K) \tag{7}$$

where K is a knowledge fact in C, p_K his probability, and $v(K)$ its associated value (e.g. expected outcome) as given by EXP agent.

[4]In real-world applications, we could ask some experienced subjects to give their rankings importance for a set of domain facts, and derive the initial salience from this. However, when we will design a learning mechanism for the salience, the importance of these initial values will be much lowered

2.3 Decision process overview

We summarize the decision process in CODAGE with the flow charts depicted in Figure 2.

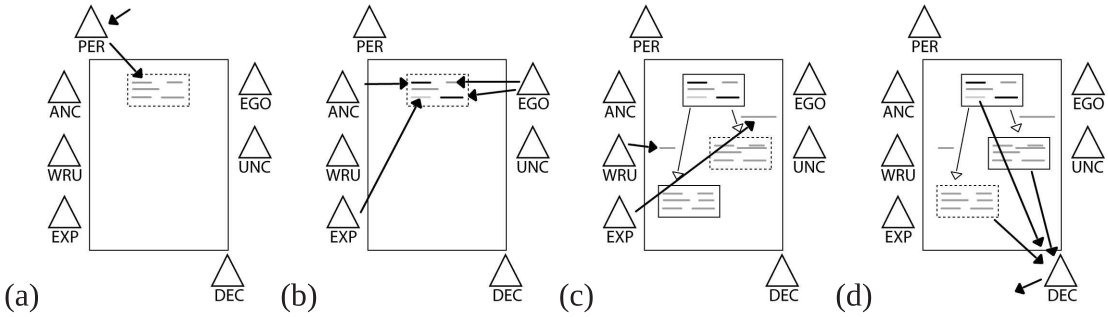

Figure 2: CODAGE decision process overview

Perception and Integration (a) The perception agent represents the current world in the root of the tree TA. (b) As soon as information appears, the EGO agent look for personal concerns and increases the corresponding saliences. Expertise agent may also update salience based on new information and its heuristics, while ANC agent increase the saliences of anchored values.

Alternative building (c) Based on the most salient facts, agents use the TA to simulate actions, and to anticipate events and other decision maker's actions in a short or medium term. New alternatives are added to the TA, from EXP, WRU and DEC among others.

Choice (d) In parallel with (b) and (c), the decision agent assesses alternatives (utility computation), apply dominance search that leads either to the choice of an action or a selection of alternatives to be further explored.

3 Simulation results

3.1 Simulating an experimental market

This architecture has been instantiated in the economy field. We did not reuse the classical benchmarks used by the *Agent-based Computationnal Economics* community, like the well-known Santa-Fe Artificial Stock Market (SF-ASM), since we want to focus on cognitive aspects of decision-making within a simulated market, while SF-ASM focus on conditions of equilibrium and market behaviors using reactive agents.

We have selected an experimental financial market conducted by Biais, Hilton, Mazurier and Pouget [Biais et al., 2004]. This experimental market is aimed to study the effects of cognitive biases on the decision of traders placed on a market under asymmetric information. On this market, traders can publish at any time buy or sell orders (fixing the count and the limit price), accept an offer or cancel a previous order. There is a single risky asset, which pays a liquidating dividend at the end of the game which can be A, B or C with equal probability (in the experiment, 50, 240 and 490). Before trading starts the players receive heterogeneous private signals. For instance, if the final dividend is B, half the participants are privately informed it is not A, while the others know it is not C. There exists no communication between participants.

Hilton et al. suggest than the participants try to analyze the actions of the others to find the final asset price. Traders are reasoning in a high-uncertainty context, making them more influenced by **cognitive biases**. The authors study two biases : *overconfidence* and *self-monitoring*. Overconfidence makes the decision-maker to overestimate the representativity of his current

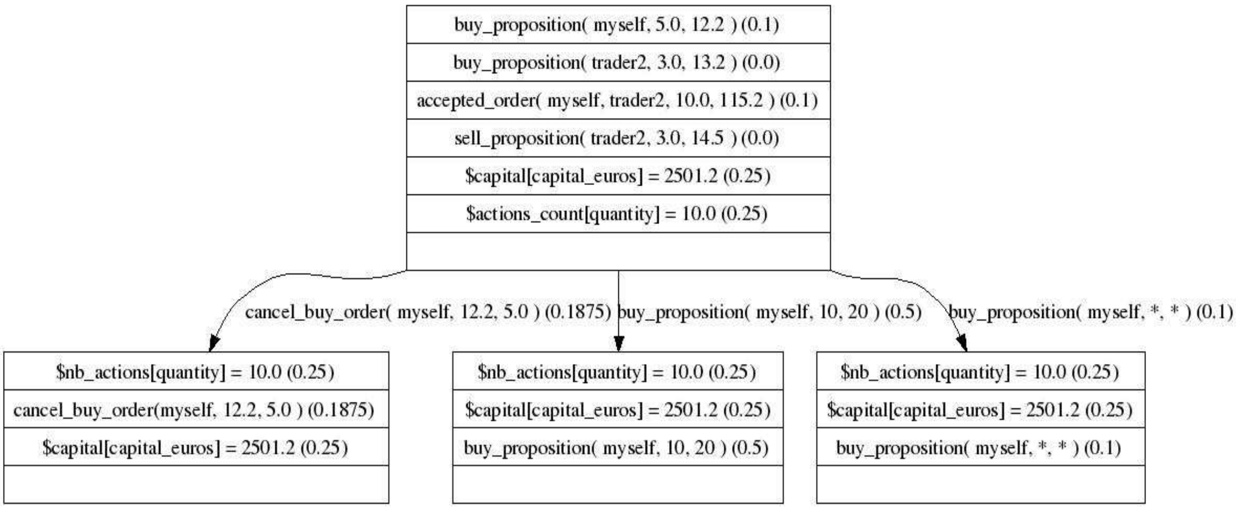

Figure 3: A Tree of Alternatives

informations. Traders suffering of self-monitoring are more attentive to the image they present to others, making them more manipulative.

We implement the **overconfidence bias** in CODAGE. In order to do so, we decrease the importance of initial probabilities (to favor current informations). Giving p_i the initial probability, $nb\_observations_K$ the number of times K is observed by the macro-agent, and $total_nb\_observations$ the total number of observations, the probability of a fact K is given by :

$$p_K = \frac{\beta.p_i + nb\_observations_K}{\beta + total_nb\_observations} \tag{8}$$

The modification of the β parameter of UNC agent modifies the sensitivity of personal experience. The self-monitoring bias seemed to be too general to be implemented yet.

The other experimental settings are as follows. We use the decision equations (7)-(8) described in section 2.2, with f set to a simple mean function, and $v(K)$ set to fixed randomly chosen values (no prior knowledge). Finally, we use here two instantiations of ANC agent : one for quantity values anchoring, and the second for prices anchoring. Each ANC agent only favor a set of discrete value (e.g. price value or quantity value), according to a salience anchoring curves like the one depicted in Figure 3.1. In this Figure, the quantity values are discretized using a step equals to 5, other values will a null salience value :

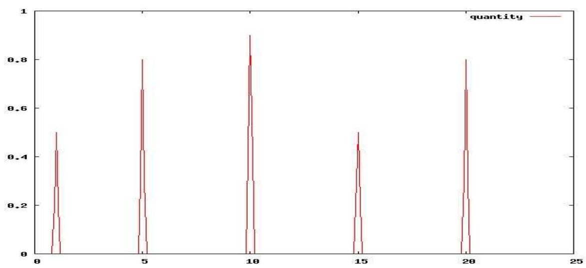

Figure 4: salience anchoring curve for quantity

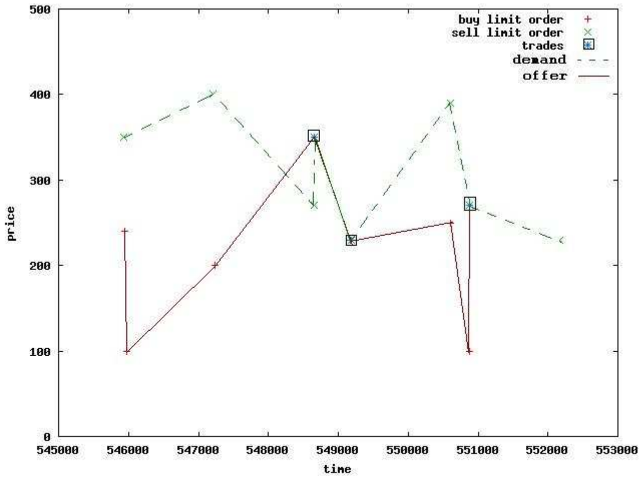

Figure 5: Biais et al. 's market simulation (extract)

3.2 Example of simulation

To see how CODAGE implements the experience described above, let us examine a tree generated by our program depicted in Figure 3 above. The process that generated this tree is the following:

- the *perception agent* added facts in the root alternative. At this time, no fact is salient, it is the raw perception.
- when a fact is added, an event "NewNonSalientFactEvent" is sent to all agents. Agents dealing with salience vote for facts : EGO agent votes for all facts concerning this trader, the ANC agents vote for salience according to their own salience curve, and the EXP agent highlights facts useful for trading (salience is displayed between brackets on the figure).
- each time a salience is modified, an event "NewSalientFactEvent" is sent with a latency, computed using Equation 5 . Each agent can react. Here, the WRU agent has proposed to cancel a previous offer or to emit a new buy order.
- when an agent proposes a new action, the TA copies salient facts (over an given recopy threshold) in the new alternative.
- WRU has added an incomplete predicate $buy\_proposition(myself, *, *)$, which contains two undefined variables : count and price. The ANC agents propose first the most salient values, here 10 unities and 20.0.
- at each alternative modification, the DEC agent evaluates - if possible - the alternative, and selects it if it is a satisficing one.

3.3 Overview of market simulation

An overview of the market simulation is displayed in Figure 5, which shows offers (plain) and demand (dotted) curves, and the trades (squares). During a primarily period, the agents put orders that are too riskless for being accepted (low offers, high demands). Then the EXP agent modifies salience of facts leading to a compromise (we supposed it was one of the trader's general heuristics). Traders will trade on this basis. Since traders use values generated by ANC agents, only anchored values will be used in the market.

4 Discussion

In this paper, we present the CODAGE approach to model human decision-making, where the decision-maker is modeled with a multi-agent system. We tried to implement concepts proposed by psychological theories. Numerical anchoring has been implemented, as selective perception. CODAGE also includes intrinsically the parcimony principle, which states that only the useful knowledge has to be processed during decision. Further work needs to be done, and among them, the ability to automatically learn the salience values is of high importance. Salience learning could be based on the fact that a salience must be high if it enhance the quality of the decision. So, if a decision has been good, we have to renforce the salience of the facts which have led to this conclusion. Otherwise, we must decrease the salience values (that lead to a bad decision). From this idea, we are currently working on a reinforcement learning algorithm for the salience values. For this simulation, we choosed an aggregative model for the DEC micro-agent. But CODAGE allows usage of any choice model (as dominance [Lee, 1971], or elimination by aspect [Tversky, 1972, page 285]) depending to the choosed simulation. Finally, the CODAGE architecture is intended to be as much generic as possible. It could be viewed as an agent-based decision framework where different decision heuristics and biases could be implemented. We should move further into that direction, and try to incorporate other models of choice, salience learning and case-based reasoning.

References

[Anderson, 1983] Anderson, J. R. (1983). *The Architecture of Cognition*. Lawrence Erlbaum Associates.

[Anderson et al., 2004] Anderson, J. R., Bothell, D., Byrne, M., Douglass, S., Lebiere, C., and Qin, Y. (2004). An integrated theory of the mind. *Psychological Review*, 111(4):1036–60.

[Axtell, 1996] Axtell, R. (1996). *Growing artifical societies: social science from the bottum up*. Washinghton, DC : Brookings Institution Press.

[Berthoz, 2003] Berthoz, A. (2003). *La Decision*. Odile Jacob, Paris.

[Biais et al., 2004] Biais, B., Hilton, D., Mazurier, K., and Pouget, S. (2004). Judgmental overconfidence, self-monitoring and trading performance in an experimental financial market.

[Brunswik, 1952] Brunswik, E. (1952). *The Conceptual Framework of Psychology*. Chicago: University of Chicago Press.

[Corkill, 2003] Corkill, D. D. (2003). Collaborating software: Blackboard and multi-agent systems & the future. In *Proceedings of the International Lisp Conference, New York*.

[Edmonds and Moss, 2001] Edmonds, B. and Moss, S. (2001). The importance of representing cognitive processes in multi-agent models. In *Artificial Neural Networks - ICANN 2001 : International Conference Vienna, Austria*.

[Engel et al., 1995] Engel, J. F., Blackwell, R. D., and Miniard, P. W. (1995). *Consumer Behaviour*. The Dryden Press, Orlando, 9 edition.

[Ferber, 1999] Ferber, J. (1999). *Multi-Agent Systems*. Addison Wesley.

[Gigerenzer and Goldstein, 1996] Gigerenzer, G. and Goldstein, D. (1996). Reasoning the fast and frugal way: Models of bounded rationality. *Psychological Review*, 103:650–669.

[Gilbert, 1994] Gilbert, N., editor (1994). *Simulating societies : the computer simulation of social phenomena*. University of Surrey, Guilford (GBR).

[Hammond et al., 1975] Hammond, K. R., Steward, T. R., Brehmer, B., and Steinmann, D. (1975). *Human Judgment and Decision Process*, chapter Social Judgment Thoery, pages 271–312. New York : Academic Press.

[Haynes and Kachelmeier, 1998] Haynes, C. and Kachelmeier, S. (1998). The effects of accounting contexts on accounting decisions: A synthesis of cognitive and economic perspectives in accounting experimentation. *Journal of Accounting Literature*, 17.

[Kahneman et al., 1982] Kahneman, D., Slovic, P., and Tversky, A., editors (1982). *Judgment under uncertainty: Heuristics and biaises*. Cambridge, England: Cambridge University Press.

[Kant and Thiriot, 2006] Kant, J.-D. and Thiriot, S. (2006). Modeling one human decision maker with a multi-agent system : the codage approach. In *Proceedings of AAMAS'06*. To appear.

[Klein, 1993] Klein, G. A. (1993). *Decision Making in Action: models and methods*, chapter A Recognition-Primed Decision (RPD) Model of Rapid Decision Making, pages 138–147. Norwood, N.J. : Ablex Pub.

[Lee, 1971] Lee, W. (1971). *Decision theory and human behavior*. New York: Wiley.

[Minsky, 1986] Minsky, M. (1986). *The society of mind*. Simon & Schuster, Inc., New York, NY, USA.

[Petrov and Anderson, 2000] Petrov, A. and Anderson, J. R. (2000). Anchor: A memory-based model of category rating. In *Proceedings of the Twenty-Second Annual Conference of the Cognitive Science Society*, pages 369–374.

[Simon, 1955] Simon, H. A. (1955). A behavioral model or rational choice. *Quarterly Journal of Economics*, 69:99–118.

[Tversky, 1972] Tversky, A. (1972). Elimination by aspects : A theory of choice. *Psychological Review*, 79:281–299.

[Tversky and Kahneman, 1986] Tversky, A. and Kahneman, D. (1986). Rational choice and the framing of decisions. *The Journal of Business*, 59(4):S251–S278.

[Wooldridge, 2002] Wooldridge, M. (2002). *An Introduction to MultiAgent Systems*. New York: J. Wiley.